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CONVECTIVE HEAT TRANSFER IN AN INTERNALLY HEATED HORIZONTAL FLUID LAYER WITH UNEQUAL BOUNDARY TEMPERATURES

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Abstract—An experimental investigation was conducted on the steady-state convective heat transfer in a horizontal water layer with isothermal, horizontal boundaries at unequal temperatures. Joule heating provided a uniform heat source throughout the fluid layer. Heat flux through each horizontal boundary was determined from the measured temperature drop across a plate of known thickness and thermal conductivity. The results were correlated by expressing the fraction of internally generated heat transferred downward as an explicit function of internal and external Rayleigh numbers.

NOMENCLATURE

c_p, specific heat at constant pressure [W s/g K];
g, gravitational acceleration [cm/s^2];
HB, heat balance, defined as $HB = (Q_1 + Q_0)/qL$;
k, thermal conductivity [W/cm K];
L, total fluid layer depth [cm];
Pr, Prandtl number, $Pr = \nu/\alpha$;
q, volumetric heat generation rate [W/cm^3];
Q_0, downward heat flux [W/cm^2];
Q_1, upward heat flux [W/cm^2];
Ra_{KG}, internal Rayleigh number as defined by Kulacki and Goldstein, $Ra_{KG} = g\beta qL^5/64\alpha\nu k$;
Ra_E, external Rayleigh number; $Ra_E = g\beta L^3(T_0 - T_1)/\alpha\nu$;
T, temperature [K];
T_0, lower boundary temperature [K];
T_1, upper boundary temperature [K].

Greek symbols

α, thermal diffusivity [cm^2/s];
β, coefficient of volumetric expansion [K^{-1}];
η, fraction of downward heat transfer in fluid layer;
η_0, value of η for $Ra_E = 0$ [see equation (3)];
ν, kinematic viscosity [cm^2/s].

Subscripts

0, evaluated at lower boundary;
1, evaluated at upper boundary.

INTRODUCTION

In an initially quiescent fluid layer, convective motion can be produced if a sufficiently large imbalance is developed between the buoyant forces, which tend to displace fluid elements, and the viscous forces, which try to maintain equilibrium. The buoyant forces which are responsible for convection may be produced in two ways; differential heating of the fluid at its horizontal boundaries, or internal heating of the fluid. The study reported here focused on steady-state convective heat transfer in the presence of internal heating in a horizontal fluid layer with rigid, isothermal, horizontal boundaries at unequal temperatures. This problem has potential applications in nuclear reactor safety analysis [1–4], as well as in geophysics [5] and astrophysics [6, 7].

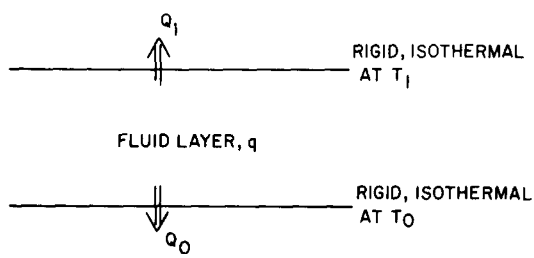

Fig. 1. Schematic representation of the heat transfer problem.

A schematic representation of the problem is shown in Fig. 1. The primary purpose of the study reported here was to correlate experimentally the fraction η of internally generated heat transferred downward as an explicit function of the internal and external Rayleigh numbers of the fluid layer. The internal Rayleigh number, as defined by Kulacki and Goldstein [8], is

$$Ra_{KG} = \frac{g\beta qL^5}{64\alpha\nu k}, \tag{1}$$

while the external Rayleigh number is defined as

$$Ra_E = \frac{g\beta L^3(T_0 - T_1)}{\alpha\nu}. \tag{2}$$

These Rayleigh numbers are two of the dimensionless groups which characterize the problem.

Several studies on convective heat transfer in internally heated layers have been reported. For the special case of equal boundary temperatures ($Ra_E = 0$), Kulacki and Goldstein [8] obtained experimental correlations for Joule heating of water ($Pr \simeq 6$) over the range $200 \leqslant Ra_{KG} \leqslant 3.8 \times 10^5$. Their results were later confirmed by Jahn and Reineke [9], who also obtained numerical solutions to the governing equations of the problem. Little work has been done, however, on internally-heated fluid layers with unequal boundary temperatures ($Ra_E \neq 0$). Suo-Anttila and Catton [10] reported some experimental data and were able to compare their results with heat-transfer rates calculated using a power-integral technique [4]. However, they did not provide a correlation of their data useful for design or analysis. Fieg [11] reported, as a part of a broader investigation, an experimental correlation for negative external Rayleigh numbers in the form

$$\frac{1-\eta}{1-\eta_0} = \left(1 + \frac{1}{32}\frac{Ra_E}{Ra_{KG}}\right)^{2.5}, \tag{3}$$

for $9 \times 10^3 \leqslant Ra_{KG} \leqslant 5 \times 10^8$ and $-6 \times 10^8 \leqslant Ra_E$ -2×10^3. The quantity η_0 refers to the corresponding value of η if $Ra_E = 0$. To our knowledge, the only existing comprehensive correlation, covering both positive and negative external Rayleigh numbers, is based on a semi-empirical model proposed by Baker *et al.* [3]. The correlation was derived from Kulacki and Emara's [12] experimental data for an internally heated fluid layer with a rigid, insulated lower boundary and a rigid, isothermal upper boundary, plus an assumption that only conductive heat transfer takes place below the plane of maximum horizontally-averaged temperature in the fluid layer. The correlation was of the form

$$\frac{(1-\eta)^{0.870}}{32\eta^2 + Ra_E/Ra_{KG}} = 0.0138 Ra_{KG}^{0.226}. \tag{4}$$

The present study was aimed in part at investigation of the validity of this semi-empirical model.

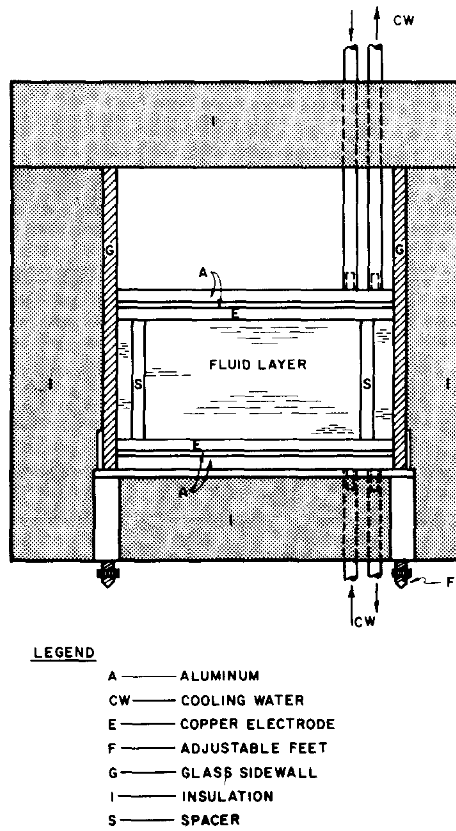

FIG. 2. Overall view of the convection cell.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

2.1. *Apparatus*

The convection cell was a layer of dilute aqueous copper sulfate solution, bounded above and below by copper plates each 25.4 cm square and 1.27 cm thick (Fig. 2). These copper plates were also the electrodes used to generate internal heating by passage of electrical current through the fluid. The fluid layer depth was varied from 1.27 to 20.32 cm by the use of phenolic-glass tubing spacers. The sidewalls were plate glass, 1.27 cm thick, held together by brackets and sealed with silicone–rubber cement. The entire convection cell was insulated thermally by a 10 cm thickness of styrofoam.

Temperatures of the horizontal boundaries were controlled by circulating water. To the back of each copper plate were bolted two aluminum plates, the first being 0.635 cm thick and flat, and the second being 1.27 cm thick with cooling channels machined into it in a double-pass pattern. For electrical insulation, a 0.0127 cm thick mylar film was placed between the copper plate and the aluminum plate adjacent to it, and all three plates were held together by eight 0.635 cm dia nylon bolts. Silicone–rubber cement was applied as a gasket between the two aluminum plates to prevent coolant leakage.

The heat flux across each horizontal boundary was calculated from the measured temperature drop between planes defined by two layers of thermocouples, one embedded in the copper plate and one embedded in the flat aluminum plate adjacent to the copper plate. Each layer consisted of nine 26-gauge thermocouples, uniformly spaced around the plate, and encased with Omega Engineering Thermcoat HT electrically-insulating cement. Thermocouple wells in the copper plates were drilled to within 0.159 cm of the surface in contact with the fluid; the thermocouples in these locations thus served to monitor the fluid boundary temperatures as well. To maintain a uniform thermal resistance, a high-thermal conductivity compound, General Electric Insulgrease 641, was applied to all contacting surfaces between the two layers of thermocouples.

Power input into the fluid was from a 110-V, 60 Hz line current, controlled by a voltage regulator rated at 0.1% accuracy. Power dissipated in the fluid was measured with a Watt transducer, with a 0.5%

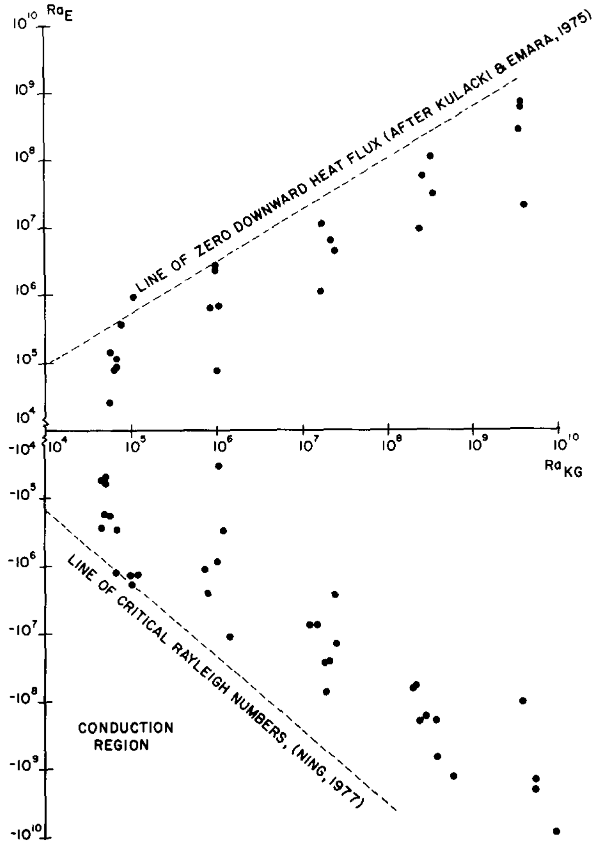

FIG. 3. Range of experimental data obtained.

rated accuracy, and a voltmeter–ammeter combination. The maximum power dissipated in the fluid was approximately 500 W.

2.2. *Procedure*

Thermal conductivity between the double layer of thermocouples in each boundary was calibrated by operating the convection cell at a 0.318 cm layer height and a power input of approximately 300 W. This operating condition resulted in an internal Rayleigh number of approximately 150, and an external Rayleigh number of approximately 100, a combination well within the conduction heat-transfer region (as defined by results of linear stability analysis [13, 14]). Heat flux through each horizontal boundary could then be determined by solution of the steady one-dimensional heat conduction equation. The known heat flux, along with the averaged measured temperature drop between the two layers of thermocouples, was then used for calculation of thermal conductivity. This calibration process was repeated regularly throughout the experiment.

Prior to each experimental run, all interior surfaces of the convection cell were cleaned thoroughly. Horizontal alignment of the lower boundary was checked with a liquid level. The cell was then filled with the copper sulphate solution to the desired height, and the upper plate lowered onto the spacers. A visual check was made through a transparent sidewall of the insulating envelope, to ensure that the upper electrode surface was uniformly in contact with the solution. Power was then applied to the cell.

For the attainment of steady-state conditions, the system was allowed to run for a period of time longer than that which would be required for achievement of steady-state conditions in heat transfer by conduction only. For the layer heights used (1.27–20.32 cm), the time required ranged from 1 to 37 h. Additional checks were made during the measurement stage to ensure that steady-state conditions had been reached before measurements were taken.

Concentration of the copper sulphate solution varied from 2.4×10^{-4} to 6.9×10^{-3} M depending on the level of power input desired. In all cases, the concentration was sufficiently low that the thermophysical properties of pure water could be assumed in all calculations. The error involved in this assumption was calculated to be less than 2% for the maximum concentration used. All fluid properties were evaluated at the arithmetic mean of the upper and lower boundary temperatures.

3. RESULTS AND DISCUSSION

3.1. *Correlation of data*

The range of experimental data obtained is shown in Fig. 3, together with the two limits for convective heat transfer in an internally heated horizontal fluid layer. The lower limit, in the region of negative external Rayleigh numbers ($T_1 > T_0$), is the line of

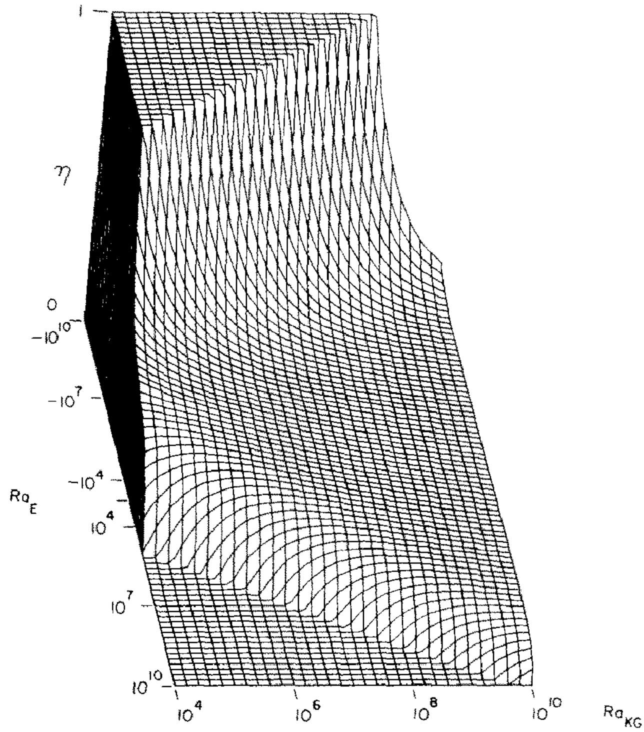

FIG. 4. A three-dimensional representation of correlation 5.

critical Rayleigh numbers as predicted by linear perturbation theory [13, 14]. The upper limit, in the region of positive external Rayleigh numbers, corresponds to the line of zero downward heat flux, as determined from Kulacki and Emara's [15] experimental data on an internally heated fluid layer with a rigid, insulated lower boundary and a rigid, isothermal upper boundary. This limiting line defines approximately the boundary between the phenomenon of an internally heated fluid layer and that of a fluid layer heated both internally and from below.

For each experimental run, a heat balance value was calculated from the relation $HB = (Q_1 + Q_0)/qL$. Any experimental run with a heat balance deviating by more than $\pm 5\%$ from unity was rejected. In all, 54 experimental runs meeting this criterion were obtained. Of this total, 42 runs were obtained with heat balances within 3% of unity.

In determining Rayleigh numbers, maximum uncertainties were estimated to be 2% for thermophysical properties, and internal heat generation rates, and 0.014 K for temperature differences. This resulted in maximum uncertainties of 5% for Ra_{KG} and 4% for Ra_E. In determining η values, the uncertainty in boundary plate conductivities was found to be about 1%. This, along with uncertainties of 0.025 K in temperature-drop measurements, led to uncertainties in Q_0 and Q_1 ranging from 1.5 to 6% and uncertainties in η ranging from 0.9 to 5.7%.

The experimental data were correlated in the form

$$\eta = -0.0695 \frac{|Ra_E|}{Ra_E} \left(\frac{|Ra_E|}{Ra_{KG}} \right)^{0.719} + 0.734 Ra_{KG}^{-0.0785} \quad (5)$$

from 54 observations, $4.7 \leqslant Pr \leqslant 7.1$, and for the range of Rayleigh numbers shown in Fig. 3. Graphical representations of this correlation are shown in Figs. 4 and 5. In both figures the correlation has been extrapolated to cover the range $0 \leqslant \eta \leqslant 1$. Maximum standard error for the correlating coefficients was 7%. Inclusion of the Prandtl number and/or the aspect ratio of the fluid layer into the model was found to produce no significant improvement in correlating the data.

3.2. *Comparison of results*

No heretofore available correlations of experimental data encompass both positive and negative values of the external Rayleigh number. Nevertheless, for certain special cases, these results may be compared to the results of other studies, namely for $Ra_E < 0$ and for heat transfer in the purely conductive regime.

For internal Rayleigh numbers less than about 2×10^5, it was possible in this study to conduct experimental runs in or very close to the conduction heat-transfer regime. The results of such experimental runs thus could be compared with the solution of the one-dimensional heat conduction equation. Although these comparisons are only approximate, they can serve as an indicator of the validity of the experimental method used. Typical results are shown in Table 1. In general, experimental values of η differed by less than 5% from values predicted for conduction heat transfer.

Kulacki and Goldstein [8] and Jahn and Reineke [9] have studied the special case of $Ra_E = 0$. Since in the present work no actual experimental data were taken at $Ra_E = 0$, any comparison made with the

Table 1. Comparisons of selected experimental results with solution of the one-dimensional conduction equation

Ra_{KG}	Ra_E	η_{exp}	η cond	Difference (%)
9.69×10^4	-1.31×10^6	0.728	0.742	−2.0
6.10×10^4	-1.28×10^6	0.862	0.828	4.1
9.91×10^4	-1.88×10^6	0.823	0.795	3.5

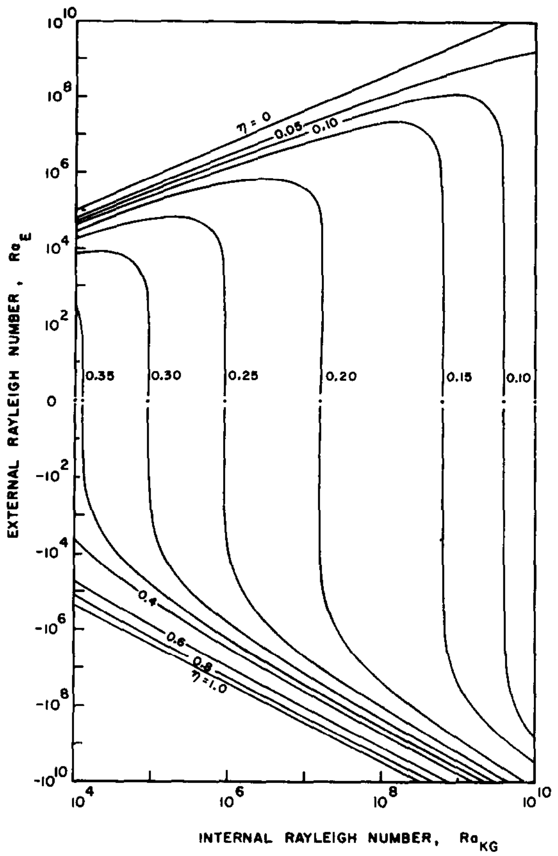

FIG. 5. A contour map of correlation 5.

two previous studies must be based on the correlation, equation (5), with Ra_E set equal to zero. The comparison illustrated in Fig. 6 shows that the present work is in general agreement with the two previous studies. At least some of the differences between correlations may be attributed to the use of different temperatures for fluid property evaluations.

In the region of negative external Rayleigh numbers, results of the present work can be compared with Fieg's correlation, equation (3). The comparison is shown in Fig. 7. The main difference between the two results is the presence of an inflection point in Fieg's correlation. This inflection point is believed due to the form of equation (3), which, although designed to yield $Ra_E/Ra_{KG} = -32$ at the limiting case of $\eta = 1.0$ consistent with the conduction solution, produces an incorrect first derivative value at $\eta = 1.0$. Specifically, equation (3) results in a first derivative

$$\left.\frac{d[(1-\eta)/(1-\eta_0)]}{d(Ra_E/Ra_{KG})}\right|_{\eta=1.0} = 0,$$

whereas a value of 1/32 is predicted by the conduction solution. The form of equation (3) also implies that the ratio $(1-\eta)/1-\eta_0)$ is solely dependent on the ratio Ra_E/Ra_{KG}, i.e., on the quantity $64k\Delta T/qL^2$; however, this could be neither confirmed nor refuted by the present investigation.

Results of this work can be compared at least qualitatively with those of Suo-Anttila and Catton

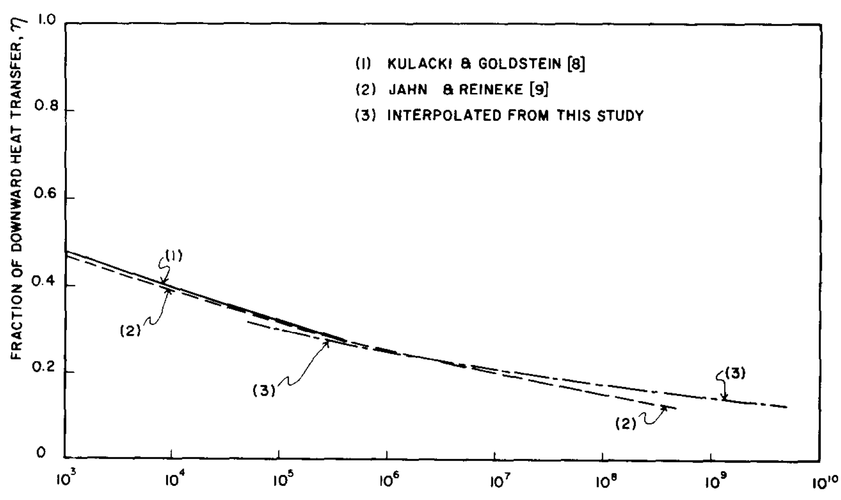

FIG. 6. Comparison of results for zero external Rayleigh numbers.

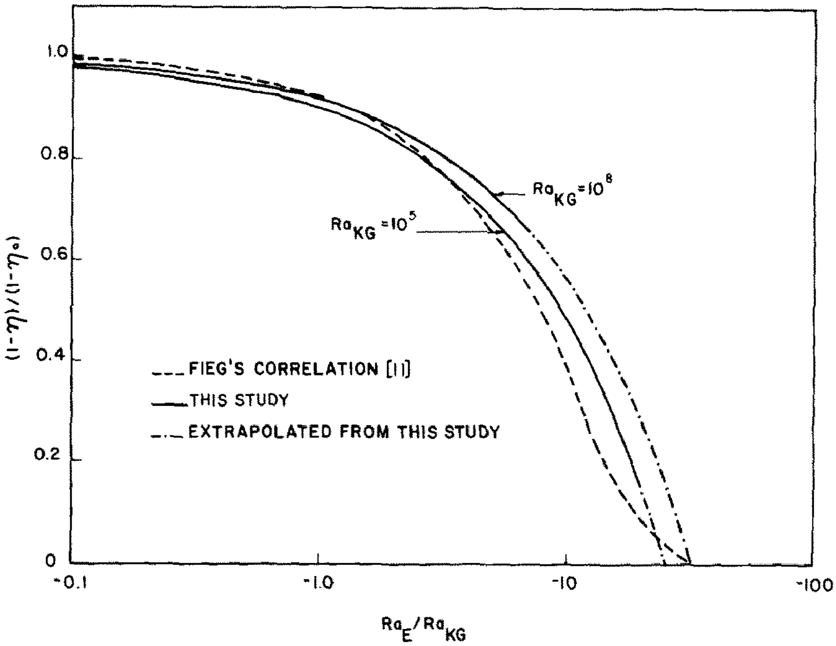

FIG. 7. Comparison of results for negative external Rayleigh numbers.

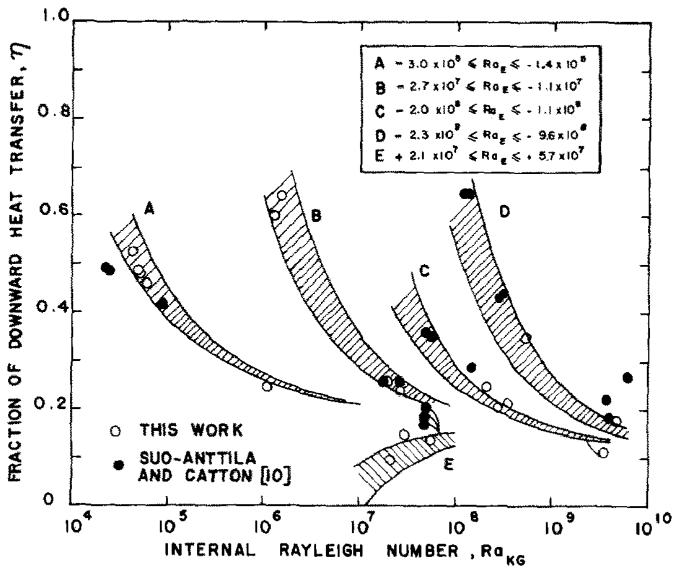

FIG. 8. Comparison of experimental data with that of Suo-Anttila and Catton [10] for selected ranges of external Rayleigh numbers. The cross-hatched regions correspond to the range of expected results as predicted by equation (5).

[4, 10]. A direct one-to-one comparison is not possible because in neither study could the external and internal Rayleigh numbers be controlled independently. It is possible to pick out from each study selected data which fall within limited ranges of external Rayleigh numbers. The comparison is shown in Fig. 8. In each group of data points, the cross-hatched region displays the range of expected results according to the correlation of equation (5). Suo-Anttila found agreement between their experimental data [10] and their theoretical power integral predictions [4]. The correlation obtained in this work is compared in Fig. 9 with the power integral predictions. The agreement is good except at large internal Rayleigh numbers.

Finally, a comparison may be made between the present work and the semiempirical model proposed by Baker *et al.*, equation (4). This is shown in Figs. 10(a) and (b), for negative and positive external Rayleigh numbers, respectively. A good agreement between the two models is observed in Fig. 10(a), in which a maximum difference of approximately 13% exists at $Ra_E = 0$ and $Ra_{KG} = 5\times10^4$. Less favorable agreement is found in Fig. 10(b), particularly for

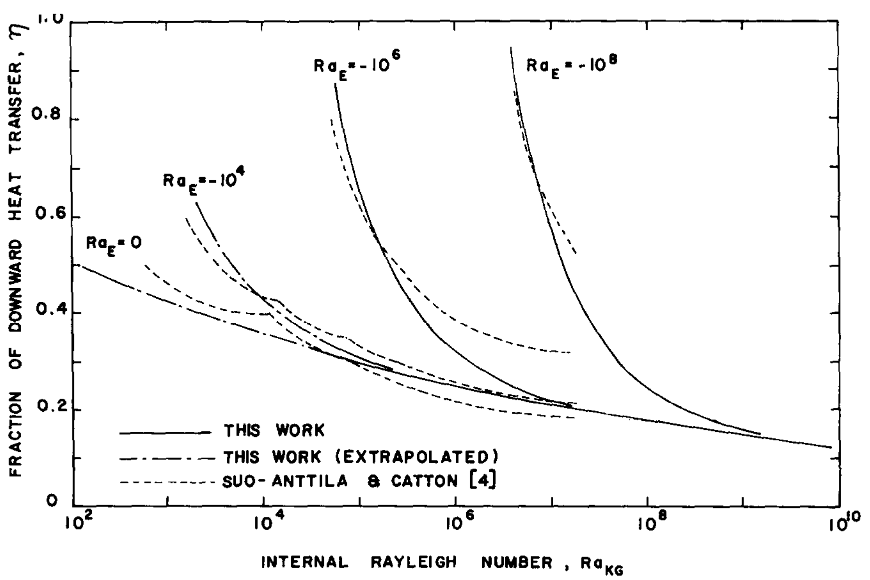

FIG. 9. Comparison of the correlations, equation (5), with theoretical power integral predictions of Suo-Anttila and Catton [4].

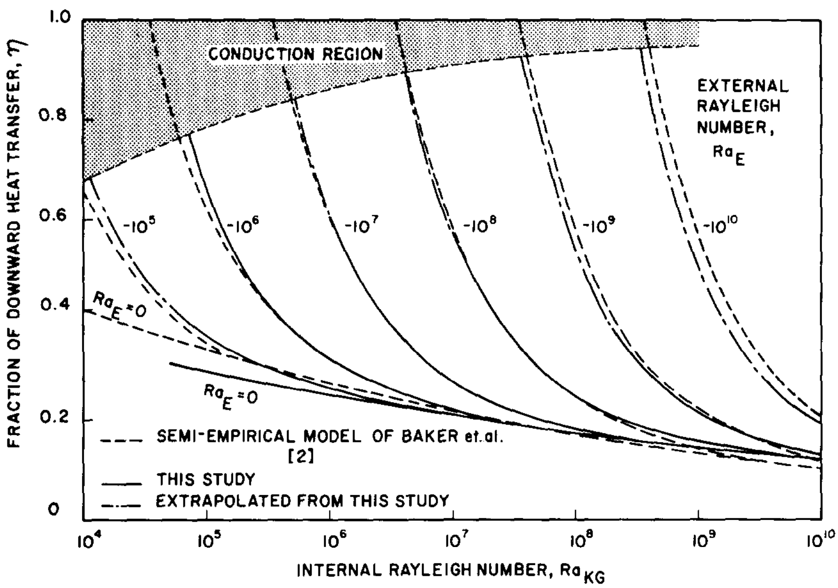

FIG. 10(a). Comparison with semi-empirical model proposed by Baker *et al.* [2], for negative external Rayleigh numbers.

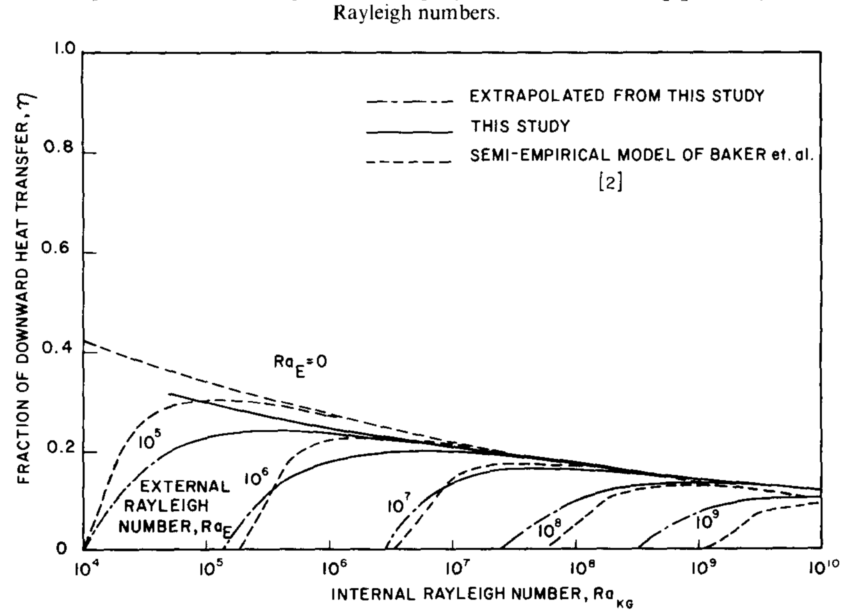

FIG. 10(b). Comparison with semi-empirical model proposed by Baker *et al.* [2], for positive external Rayleigh numbers.

both Rayleigh numbers less than approximately 10^7. Part of the difference again is attributed to the use of different temperatures for fluid property evaluations. The present study used the arithmetic mean between the upper and lower boundary temperatures, while the Baker model was based partly on Kulacki and Emara's [12] experimental data, for which fluid properties were evaluated at the upper boundary temperature.

3.3. *Conclusion*

This study has resulted in a correlation by which may be predicted the magnitude of downward heat transfer in an internally heated fluid layer with isothermal boundaries at unequal temperatures. The experimental correlation obtained is valid for both the positive and negative external Rayleigh numbers, and is based on extensive experimental data obtained under carefully controlled conditions. The correlation has the practical advantage of expressing η as an explicit function of Ra_{KG} and Ra_E. The range of Rayleigh numbers investigated is sufficiently broad that the correlation should find wide practical application.

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CONVECTION THERMIQUE DANS UNE COUCHE FLUIDE HORIZONTALE AVEC CREATION DE CHALEUR INTERNE ET AVEC DES TEMPERATURES DIFFERENTES AUX FRONTIERES

Résumé—On étudie expérimentalement la convection thermique, en régime permanent, dans une couche d'eau horizontale avec des températures uniformes mais différentes sur chaque frontière horizontale. Le chauffage par effet Joule est fourni par une source de chaleur uniforme à travers la couche de fluide. Le flux de chaleur à chaque frontière horizontale est déterminé à partir de la mesure de la chute de température à travers une plaque d'épaisseur et de conductivité thermique connues. Les résultats sont représentés par une formule qui exprime explicitement la fraction de chaleur générée qui est transférée vers le bas, en fonction des nombres de Rayleigh interne et externe.

KONVEKTIVE WÄRMEÜBERTRAGUNG IN EINER VON INNEN BEHEIZTEN HORIZONTALEN FLÜSSIGKEITSSCHICHT BEI UNTERSCHIEDLICHEN RANDTEMPERATUREN

Zusammenfassung—Zur stationären konvektiven Wärmeübertragung in einer horizontalen Wasserschicht mit isothermen horizontalen Rändern wurden experimentelle Untersuchungen bei unterschiedlichen Temperaturen durchgeführt. Mittels elektrischer Beheizung wurde eine gleichförmige Wärmequelle in der gesamten Flüssigkeitsschicht erzeugt. Aus dem gemessenen Temperaturgefälle an einer Flüssigkeitsschicht bekannter Dicke und Wärmeleitfähigkeit wurde der Wärmestrom durch jede horizontale Granzfläche bestimmt. Die Ergebnisse wurden dargestellt, indem das Verhältnis der im Inneren erzeugten Wärme zu der nach unten übertragenen als explizite Funktion der inneren und äußeren Rayleigh-Zahlen ausgedrückt wurde.

КОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС В ГОРИЗОНТАЛЬНОМ СЛОЕ ЖИДКОСТИ С ВНУТРЕННИМИ ИСТОЧНИКАМИ ТЕПЛА И РАЗЛИЧНЫМИ ТЕМПЕРАТУРАМИ НА ГРАНИЦАХ

Аннотация — Проведено экспериментальное исследование стационарного конвективного переноса тепла в горизонтальном слое воды с горизонтальными изотермическими границами, имеющими различную температуру. Джоулев нагрев обеспечивал равномерно распределенный источник тепла по всему слою. Тепловой поток через каждую горизонтальную границу определялся по измеренному перепаду температуры на пластине известной толщины и с известной теплопроводностью. Результаты обобщены в виде зависимости, в которой доля генерируемого внутри слоя тепла, переносимая книзу, является явной функцией чисел Релея внутри и вне слоя.